Santee School District

MATHEMATICS PROFESSIONAL DEVELOPMENT

Grade One

December 5, 2013

MENTAL MATH

During recess there are 98 students on the playground. 87 students join them. How many students are now on the playground?

Record how you mentally solved this problem.

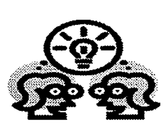
Mathematically Productive Teaching Routine
Structuring Student Math-Talk

Purposes
- Support the development of student-to-student interaction that is consistently equitable, status-free, and mathematically productive
- Provide formative assessment information that drives instructional decisions

Student Outcomes
- Equitable, status-free, and mathematically productive student-to-student interaction
- Increased metacognitive skills
- Increased capacity to articulate and clarify their math thinking
- Increased math content knowledge
- Improved Mathematical Habits-of-Mind
- Increased accountability and engagement
- Increased self-efficacy as mathematicians

Structures

When students work in a Mathematicians Dyad, Triad, or Quad, the math-talk:
- <u>Always begins</u> with "Mathematicians Think Time" (i.e., time to think privately) about the task
- <u>Always focuses</u> on each group member's mathematical reasoning, sense making, representations, justifications, and/or generalizations
- <u>Always ends</u> with a discussion of ways their ideas are mathematically the same and/or different
- <u>Always follows</u> a prescribed structure that provides students "practice" with status-free, and mathematically productive student-to-student interaction

<table>
<tr><td>LISTEN & COMPARE
</td><td>A. Partner #1 explains her/his ideas while the other partner(s) silently listen to understand Partner #1's thinking.
B. When the teacher announces, "Finish your thought and switch roles," repeat step A for question/task and student backgrounds.
C. (for triads and quads) Repeat until all partners have reported.</td></tr>
<tr><td>REVOICE & COMPARE
</td><td>A. Partner #1 speaks while the other partner(s) silently listen to understand Partner #1's mathematical thinking.
B. When the teacher announces, "Finish your thought and Partner #X revoice," Partner #X carefully revoices Partner #1's ideas without judging, adapting, or commenting about the correctness or sensibility of the ideas.
C. Partner #1 clarifies as needed.
D. When the teacher announces, "Rotate Partners," Partner #2 speaks while the other partner(s) silently listen to understand.
E. When the teacher announces, "Finish your thought and Partner #Y revoice," Partner #Y carefully revoices Partner #2's ideas.
F. Partner #2 clarifies as needed.
G. (for triads and quads) Repeat until all partners have revoiced and reported.</td></tr>
<tr><td>INTERPRET & COMPARE
</td><td>A. Two partners exchange their written work for a task. During Private Think Time, the partners study each other's work and, without any discussion, try to understand each other's reasoning.
B. Partner #1 reports her interpretation of Partner #2's reasoning.
C. Partner #2 clarifies.
D. Partner #2 reports his interpretation of Partner #1's reasoning.
E. Partner #1 clarifies.</td></tr>
</table>

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,

communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Mathematics | Grade 1

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

(1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

(2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

(3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[1]

(4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

[1]Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

Grade 1 Overview

Operations and Algebraic Thinking

- **Represent and solve problems involving addition and subtraction.**
- **Understand and apply properties of operations and the relationship between addition and subtraction.**
- **Add and subtract within 20.**
- **Work with addition and subtraction equations.**

Number and Operations in Base Ten

- **Extend the counting sequence.**
- **Understand place value.**
- **Use place value understanding and properties of operations to add and subtract.**

Measurement and Data

- **Measure lengths indirectly and by iterating length units.**
- **Tell and write time.**
- **Represent and interpret data.**

Geometry

- **Reason with shapes and their attributes.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 1.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]

2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between addition and subtraction.

3. Apply properties of operations as strategies to add and subtract.[3] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*

4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

Add and subtract within 20.

5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Work with addition and subtraction equations.

7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ▯ – 3, 6 + 6 = ▯.*

Number and Operations in Base Ten 1.NBT

Extend the counting sequence.

1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

   a. 10 can be thought of as a bundle of ten ones — called a "ten."

   b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

   c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

[2]See Glossary, Table 1.
[3]Students need not use formal terms for these properties.

3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Use place value understanding and properties of operations to add and subtract.

4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Measurement and Data 1.MD

Measure lengths indirectly and by iterating length units.

1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.

2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*

Tell and write time.

3. Tell and write time in hours and half-hours using analog and digital clocks.

Represent and interpret data.

4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Geometry 1.G

Reason with shapes and their attributes.

1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

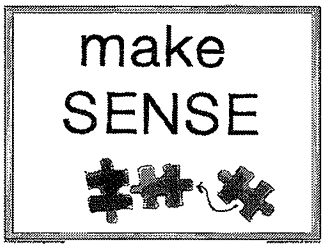
2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[4]

3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

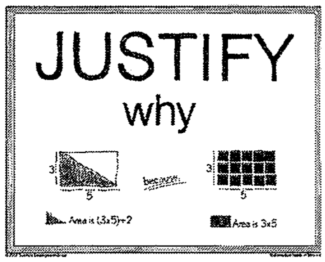
[4]Students do not need to learn formal names such as "right rectangular prism."

Name: **Date:**

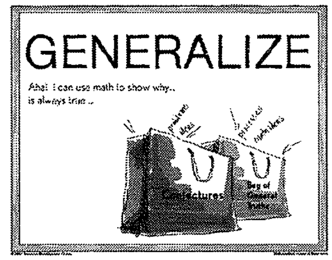
STUDENT REFLECTION TOOL: MATHEMATICAL HABITS OF MIND



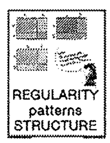
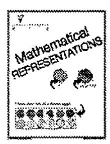
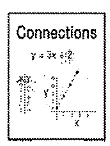
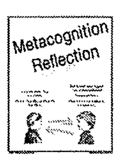
To **make sense** of math ideas and problems, I look for *regularity, patterns, structure, representations, connections, and other math I know*. I *reflect* about my own and others' thinking and *mistakes* and I *persevere* to be sure that ideas and problems make sense.



Math ideas and solutions make sense when I can use *regularity, patterns, structure, representations, connections, and other math I know* to **justify** why the ideas and solutions are always, sometimes, or never true.



I use *regularity, patterns, structure, representations, connections, and other math I know* to make **conjectures** about math ideas I think are always, sometimes, or never true. I create **mathematical generalizations** by justifying why conjectures are valid.

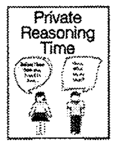
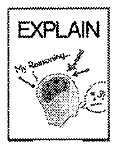
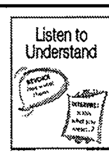
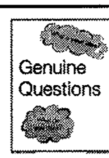
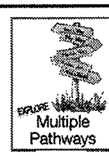
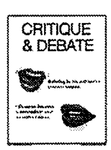
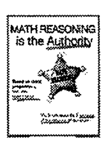
To make **S**ense of math ideas and problems, and to support **C**onjectures, **J**ustifications, and **G**eneralizations:	**S, C, J, G**	**Evidence**
I notice and reason about mathematical REGULARITY in repeated reasoning, PATTERNS, and STRUCTURE (meanings, properties, definitions). 		
I create and reason from MATHEMATICAL REPRESENTATIONS – visual models, graphs, numbers, symbols and equations, and situations. 		
I notice and reason about CONNECTIONS within and across mathematical representations, other math ideas, and everyday life. 		
I explore MISTAKES and STUCK POINTS to start new lines of reasoning and new math learning. 		
I use METACOGNITION and REFLECTION. I think about my math reasoning and disequilibrium – how my thinking is changing and how my ideas compare to other mathematicians' ideas. 		
I PERSEVERE and SEEK MORE. I welcome challenging math problems and ideas, and after I figure something out, I explore new possibilities. 		

Name: **Date:**

STUDENT REFLECTION TOOL: MATHEMATICAL HABITS OF INTERACTION

Use the following scale to rate your use of the Mathematical Habits of Interaction.

0	1	2	3	4
Never/Hardly ever		*Sometimes*		*It's just how I do math!*

When I do math with other mathematicians, we:		Rating	Evidence
Honor each other's right to PRIVATE REASONING TIME before talking about our ideas.			
EXPLAIN how we think and reason mathematically.			
LISTEN TO UNDERSTAND each other's math reasoning about problems, conjectures, justifications, and generalizations.			
Use GENUINE QUESTIONS to inquire about each other's math reasoning about problems, conjectures, justifications, and generalizations.			
Explore MULTIPLE PATHWAYS by applying each other's lines of reasoning.			
COMPARE our math LOGIC and IDEAS to figure out how they are mathematically the same and different.			
CRITIQUE and DEBATE the math logic and truth in each other's reasoning.			
Use MATH REASONING as the AUTHORITY for deciding what is correct and makes sense.			

Hess' Cognitive Rigor Matrix & Curricular Examples: Applying Webb's Depth-of-Knowledge Levels to Bloom's Cognitive Process Dimensions – *M-Sci*

Revised Bloom's Taxonomy	Webb's DOK Level 1 Recall & Reproduction	Webb's DOK Level 2 Skills & Concepts	Webb's DOK Level 3 Strategic Thinking/ Reasoning	Webb's DOK Level 4 Extended Thinking
Remember Retrieve knowledge from long-term memory, recognize, recall, locate, identify	o Recall, observe, & recognize facts, principles, properties o Recall/ identify conversions among representations or numbers (e.g., customary and metric measures)			
Understand Construct meaning, clarify, paraphrase, represent, translate, illustrate, give examples, classify, categorize, summarize, generalize, infer a logical conclusion (such as from examples given), predict, compare/contrast, match like ideas, explain, construct models	o Evaluate an expression o Locate points on a grid or number on number line o Solve a one-step problem o Represent math relationships in words, pictures, or symbols o Read, write, compare decimals in scientific notation	o Specify and explain relationships (e.g., non-examples/examples; cause-effect) o Make and record observations o Explain steps followed o Summarize results or concepts o Make basic inferences or logical predictions from data/observations o Use models /diagrams to represent or explain mathematical concepts o Make and explain estimates	o Use concepts to solve non-routine problems o Explain, generalize, or connect ideas using supporting evidence o Make and justify conjectures o Explain thinking when more than one response is possible o Explain phenomena in terms of concepts	o Relate mathematical or scientific concepts to other content areas, other domains, or other concepts o Develop generalizations of the results obtained and the strategies used (from investigation or readings) and apply them to new problem situations
Apply Carry out or use a procedure in a given situation; carry out (apply to a familiar task), or use (apply) to an unfamiliar task	o Follow simple procedures (recipe-type directions) o Calculate, measure, apply a rule (e.g., rounding) o Apply algorithm or formula (e.g., area, perimeter) o Solve linear equations o Make conversions among representations or numbers, or within and between customary and metric measures	o Select a procedure according to criteria and perform it o Solve routine problem applying multiple concepts or decision points o Retrieve information from a table, graph, or figure and use it solve a problem requiring multiple steps o Translate between tables, graphs, words, and symbolic notations (e.g., graph data from a table) o Construct models given criteria	o Design investigation for a specific purpose or research question o Conduct a designed investigation o Use concepts to solve non-routine problems o Use & show reasoning, planning, and evidence o Translate between problem & symbolic notation when not a direct translation	o Select or devise approach among many alternatives to solve a problem o Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze Break into constituent parts, determine how parts relate, differentiate between relevant-irrelevant, distinguish, focus, select, organize, outline, find coherence, deconstruct	o Retrieve information from a table or graph to answer a question o Identify whether specific information is contained in graphic representations (e.g., table, graph, T-chart, diagram) o Identify a pattern/trend	o Categorize, classify materials, data, figures based on characteristics o Organize or order data o Compare/ contrast figures or data o Select appropriate graph and organize & display data o Interpret data from a simple graph o Extend a pattern	o Compare information within or across data sets or texts o Analyze and draw conclusions from data, citing evidence o Generalize a pattern o Interpret data from complex graph o Analyze similarities/differences between procedures or solutions	o Analyze multiple sources of evidence o analyze complex/abstract themes o Gather, analyze, and evaluate information
Evaluate Make judgments based on criteria, check, detect inconsistencies or fallacies, judge, critique			o Cite evidence and develop a logical argument for concepts or solutions o Describe, compare, and contrast solution methods o Verify reasonableness of results	o Gather, analyze, & evaluate information to draw conclusions o Apply understanding in a novel way, provide argument or justification for the application
Create Reorganize elements into new patterns/structures, generate, hypothesize, design, plan, construct, produce	o Brainstorm ideas, concepts, or perspectives related to a topic	o Generate conjectures or hypotheses based on observations or prior knowledge and experience	o Synthesize information within one data set, source, or text o Formulate an original problem given a situation o Develop a scientific/mathematical model for a complex situation	o Synthesize information across multiple sources or texts o Design a mathematical model to inform and solve a practical or abstract situation

Student Mathematical Discourse Types

Discourse Types	Examples from Case Study
PROCEDURES/FACTS *No evidence of reasoning.* • Short answer to a direct question • Restating facts/statements/rules • Showing or asking for procedures *Uses meanings, definitions, properties, known math ideas to describe reasoning when:* • Explaining ideas and methods • Questioning to clarify • Noticing relationships/connections • **But *doesn't show why* the ideas/methods work**	
JUSTIFICATION *Reasons with meanings of ideas, definitions, math properties, established generalizations to:* • Show why an idea/solution is true • Refute the validity of an idea • Give mathematical defense for an idea that was challenged	
GENERALIZATION *Reasons with math properties, definitions, meanings of ideas, established generalizations, and mathematical relationships as the basis for:* • Making conjectures about what might happen in the general or special cases Or • Justifying a conjecture about what will happen in the general or special cases	

Finding a Missing Change

Daisy

Grade 1, December

My class has been very receptive to exploring mathematical concepts this year. Recently, I decided to explore missing addends to see if that leads us into thinking about how addition and subtraction are related.

I presented the problem:

> There were 5 students in the classroom. Some more students walked in. Now, there are 7. How many students walked in?

I asked the students to think about the problem for a few minutes before we began the discussion.

Teacher: So, how many students walked in?

Andrew: 8

Teacher: Why?

Andrew: 5 and 7 is 8.

Teacher: Can you show us how 5 and 7 is 8?

Andrew held up his fingers showing 5 on one hand and 3 on the other (6, 7, 8). I was very confused about what Andrew was thinking. He was very matter of fact with his answer. Could he have been counting up 1 more past 7 to get 8?

Teacher: I am confused how you got your answer.

Andrew: See, if you have 5 here, then 3, there's 8. So the answer is 8.

I decided to move on to see if another child's response would help Andrew. I am still not sure what Andrew was actually doing.

Ellie: 12. 12 kids came in. There's 5, then 7, so that is 12.

Teacher: Ellie, I want you to read the problem to us again.

Ellie is a good reader, so I felt comfortable asking her to read the problem on the chart to the class.

Ellie: There were 5 students in the classroom. Some more students walked in. Now, there are 7. How many students walked in? See, Mrs. Price, first there is 5, then 7. So there's 12.

I saw that Ellie was looking at the two numbers and immediately combining them. I do not think that Ellie was thinking about what was going on in the problem. I asked the other students if there were any different answers.

Derek: Yeah, 5 + 2 = 7. So, 2 kids walked in.

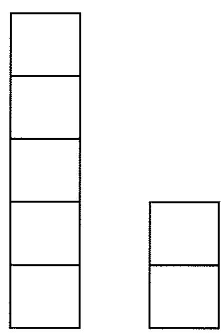
Teacher: Can you demonstrate what you mean?

Derek: Sure. See there are 5 kids. (Derek gets 5 kids to stand up by the door). Two kids walked in. (He gets 2 more kids and takes them out into the hall, then walks them back in the room). Now there are 7. See? 1, 2, 3, 4, 5, 6, 7.

Class: Ohhhhhhhhhh.

Teacher: Derek, can you show u with cubes?

Derek made a stack of 5 cubes to show the students who were first in the classroom and a stack of 2 cubes to show the students who walked in, making a total of 7.



At this time I decided to write out a number sentence for the class.

$$5 + \square = 7$$

Teacher: Is this a number sentence for our problem?

Ellie: Now, put the 2 in the box. That works.

Andrew: It's like the box is holding the 2 kids that walked in.

I decided to present the next problem.

There are 7 kids in the classroom. Some kids left. Now there are 5 kids in the class. How many kids left the classroom?

Again, I asked the students to think about what was happening in the problem. I waited a few minutes and then read the problem looking for a response to the question, "How many kids left the classroom?"

Alex: 2

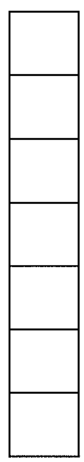
Teacher: How do you know?

Alex: If you take 7 out, there would be 0. But, when you take 2 out there would be 5 left.

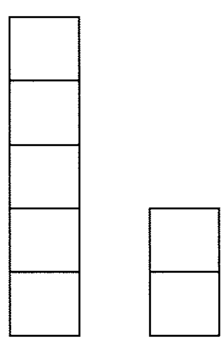
Teacher: Can you show me with cubes?

Alex put 7 cubes in a tower.

Alex: Pretend there are 2 bus kids that left the room. Now, there are 5 walkers left with you. See, take 2 away.



Alex took 2 cubes away and was left with this result.



Derek stood up and pointed to himself.

Teacher: Why are you pointing to yourself, Derek?

Derek: I'm a bus kid. I left.

I think he was trying to make a connection to the problem.

I was very pleased to see that the cube representations from the two problems looked so similar. Although the actions to get to the end result were different, the representations still resembled each other.

I held up the two cube representations.

Teacher: Does anyone notice anything about these cubes?

Derek: It's the same, but on this one, 2 came in and on that one, 2 left.

Alex: 2 came in, which gave us more. Then 2 left, so we took them away.

I was very happy that Alex was able to verbalize the actions that were taking place. My students have been having difficulty lately describing what actually happens in a given problem in context. At times, they can repeat the story to me, but they have been having trouble associating an action to the problem.

Teacher: So, what is the same about these problems?

Kayana: The numbers are the same.

Andrew: Yea, 2 came in; then 2 left. 5 were there, and 7 started there.

Alex: 5 + 2 = 7 and 7 – 2 = 5. They are the same thing but turned around. See? They are the same, but 7 -2 = 5 has a take away, and the 5 + 2 has a plus.

What a great way to end a lesson! I felt very comfortable that my students recognized and understood the relationship between the two problems. I wonder if they would be able to make the same connection to a similar pair of problems that involved different numbers. I am sure they would be able to walk through the problems the way we did in this example; but would my students be able to use the connection they had already made and apply it to a new problem? I think it goes with the question, Will it *always* work? Does each addition problem have a corresponding subtraction problem? I think that is something I will explore with my students in the future.

Name ______________________________________

There were 5 students in the classroom.
Some more students walked in.
Now, there are 7 kids in the classroom.
How many students walked in?

Name ___________________________________

There are 7 kids in the classroom.
Some kids left.
Now there are 5 kids in the classroom.
How many kids left the classroom?

Student Discourse Observation Tool

Scripting of Student Discourse	Discourse Type P/F, J, or G

Student Discourse Observation Tool

Scripting of Student Discourse	Discourse Type P/F, J, or G

Classroom Observation - Reflection

1.
What mathematical ideas did students seem to understand? What is your evidence?

2.
With what mathematical ideas were students struggling? What is your evidence?

3.
How would you characterize the students' mathematical discourse?

Commitments

1. Read the article "3 Ways that Promote Student Reasoning."

2. Work problems 1b and 2b in the article. Mathematically justify your thinking.

3. Conduct a Structured Math Talk with your students a minimum of one time per week.

4. Bring successes and challenges to our next session.